Perhaps, the process of factoring by removing the greatest common factor can be best stated as the **reverse distributive property**. In the distributive property, one is **multiplying** a certain factor to all of the terms. In factoring by **GCF**, one is **dividing** all of the terms by the **GCF**.

Consider this expression which utilizes the distributive property: $5x^2 (4x^4 + 3)$.

Visually, this is the distributive process: $5x^2 (4x^4 + 3)$.

To simplify using the distributive property, one multiplies $5x^2$ times $4x^4$, and then one multiplies $5x^2$ times 3.

After simplifying using the distributive property, you get $20x^6 + 15x^2$.

This section will now demonstrate how to factor by removing the **GCF**.

Let's now take your answer to the problem above: $20x^6 + 15x^2$.

Using what was learned in the last lesson, the **GCF** of $20x^6$ and $15x^2$ is $5x^2$. Recall - this is because the greatest common factor of 20 and 15 is 5, and because the **GCF** of like variable quantities is always the lowest exponent.

Now, **divide** each term in the original expression by the **GCF** ($5x^2$). Divide $20x^6$ by $5x^2$, and divide $15x^2$ by $5x^2$.

Therefore, after dividing by the **GCF**, the expression is $4x^4 + 3$.

To complete this **reverse distributive process**, write the **GCF** in front of a set of parentheses. Inside of the parentheses, place the expression that is left after dividing by the **GCF**.

$$= \frac{5x^2 (4x^4 + 3)}{GCF \text{ what's left after dividing}}$$

So, after factoring by removing the **GCF**, the answer is $5x^2 (4x^4 + 3)$. Note how this is the original question before distributing at the very top of the page.

Factor the greatest common factor: $8y^5 - 12y^3 + 4y$.

The **GCF** is of the three terms is $4y$, because the **GCF** of 8, 12, and 4 is 4, and the **GCF** of $y^5$, $y^3$, and $y$ is $y$. So, the **GCF** ($4y$) will be placed in front of the parentheses, and all of the terms in the expression will be divided by $4y$. 
\[
8y^5 - 12y^3 + 4y \\
+ 4y \
+ 4y \
+ 4y \\
= 4y (2y^4 - 3y^2 + 1)
\]

Therefore, the answer is \[4y(2y^4 - 3y^2 + 1)\].

Generating the last term in this expression is where many students make a mistake. In order to get "+1", one has to divide \(4y\) by \(4y\). Some students would think this is zero, and they would not write anything. However, it's important to see that \(4y \div 4y = 1\).

Factor the greatest common factor: \(14z^8 + 24z^7 - 30z^3\).

First, the \(GCF\) of all three terms is \(2z^3\). Now, divide each of the terms by \(2z^3\).

\[
14z^8 + 24z^7 - 30z^3 \\
+ 2z^3 \
+ 2z^3 \
+ 2z^3 \\
= 2z^3 (7z^5 + 12z^4 - 15)
\]

The answer is \[2z^3(7z^5 + 12z^4 - 15)\].

Factor the greatest common factor: \(16c^7 - 6c^3\).

The \(GCF\) is \(2c^3\). Now, you complete the problem below:

\[
16c^7 - 6c^3 \\
+ + \\
\downarrow \downarrow \\
\frac{GCF}{what's \ left \ after \ dividing} (\quad - \quad)
\]

For Questions 1-2, factor the greatest common factor.

1. \(25d^5 + 45d^4\)  
2. \(9k^4 + 12k^3 - 6k\)
Factor the greatest common factor: \(28a^3b^2 - 36a^2 - 17b^5\).

Note that the GCF of the coefficients (28, -36, and -17) is 1. Also, note that the terms do not all share any common variables.

Obviously, it makes little sense to write \(1(28a^3b^2 - 36a^2 - 17b^5)\).

When one is only factoring out the greatest common factor, and the GCF is 1, he/she should write that the expression is **PRIME**.

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**Homework on Factoring by Greatest Common Factor**

Factor the greatest common factor out of the polynomial. If the GCF is 1, write **PRIME**.

1. \(8x^2 + 10x\)  
2. \(12y - 16\)  
3. \(-15d^5 + 45d^3\)

4. \(13a + 20b\)  
5. \(c^3 + c^2 - c\)  
6. \(6n^2 - 30n + 42\)

7. \(-7m^2 - 10m + 17\)  
8. \(18p^3 - 63p^2 - 9p\)  
9. \(18x^2 - 50y^2\)

10. \(100z^9 + 50z^6 - 75z^5\)  
11. \(36rs^2 - 108r^2s^3\)  
12. \(36k - 30\)

13. \(a^7b - a^{10}\)  
14. \(2c^5d^4 - 3c^4 + 4c^3\)  
15. \(3g^8 + 3g^7\)

16. \(18x^5 - 48x^4 + 56x^3 - 86x\)  
17. \(23y^{10} - 46y^7 + 68y^2 + 10y\)

---

1. \(2x(4x + 5)\)  
2. \(4(3y - 4)\)  
3. \(15d^3(-d^2 + 3)\) or \(-15d^3(d^2 - 3)\)

4. **PRIME**  
5. \(c(c^2 + c - 1)\)  
6. \(6(n^2 - 5n + 7)\)  
7. **PRIME**

8. \(9p(2p^2 - 7p - 1)\)  
9. \(2(9x^2 - 25y^2)\)  
10. \(25z^5(4z^4 + 2z - 3)\)

11. \(36rs^2(1 - 3rs)\)  
12. \(6(6k - 5)\)  
13. \(a^7(b - a^3)\)

14. \(c^3(2c^2d^4 - 3c + 4)\)  
15. \(3g^7(g + 1)\)  
16. \(2x(9x^4 - 24x^3 + 28x^2 - 43)\)

17. \(y(23y^9 - 46y^6 + 68y + 10)\)